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AUTHOR Kelly, Brendan
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ABSTRACT

The author outlines common characteristics of the content and style of mathematics programs during three distinct periods of this century. He also comments upon the general restrictions imposed by publishers upon the quality of mathematics textbooks and offers some conjectures as to the nature of mathematics, curricula, instructional materials, and evaluative procedures in the 1980s. (MN)

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TWENTIETH-CENTURY REVOLUTIONS IN MATHEMATICS EDUCATION

by Brendan Kelly

1. Introduction

Mathematics education more than any other subject area, has undergone a series of profound revolutions during the latter half of this century. The dramatic impact of the sputnik launch played a catalytic role in ushering in the new math regime. Subsequent public disillusionment with the conceptual approach and the corresponding back to basics backlash generated a counter current in the mainstream of mathematics education and sparked a skills vs. concept controversy among mathematics educators. The uprooting transition to metric units and the shift into a decimal approach to arithmetic has been occurring at different rates throughout the country and threatens to break the continuity of mathematics education on a nation-wide scale. The advent of the hand-held calculator and computer-assisted instruction have brought additional complications to the mathematics scene. All these revolutions and changes have influenced significantly the curriculum content, instructional materials and evaluation procedures of contempory mathematics programs. In this paper, we review some of these implications in light of past revolutions and present trends. From the past and present perspectives we offer some conjectures as to the nature of mathematics, curricula, instructional materials and evaluative procedures in the 1980's.

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2. An Historical Overview

2.1 Mathematics Education 1900-1949

One of the major roles of education is to teach children the skills and concepts they will need to function effectively in society. Hence, the content of a mathematics program should relate to the existing social structure and technology. In the first half of the 20th Century, "mathematics" meant "arithmetic" to all but a small percentage of the population. Computational skill was crucial to the proprietor of the corner-store, the local fruit merchant, the accountant, and almost anyone involved in business. Changing ounces to pounds, board feet to cords, and pints to hogsheads were part of a daily routine. Computational skill was as crucial to the bank teller in balancing accounts as the rapid conversion of chains to miles was to the surveyor. Algebra, on the other hand, was accessible to few and was regarded by many as an esoteric form of mathematics with as little practical value as Latin or Greek. Calculus was considered an abstruce mathematical discipline to be mastered by only a few of the elite who attended university.

The focus on arithmetic was natural and appropriate to the job market of that period. The hundreds of hours spent on drilling computational skills, memorizing conversions from yards to rods and miles to inches, were warranted by the daily demand for computational facility. This High School Entrance Examination, which was administered by the Ontario Department of Education in 1903, reflects the mathematics curriculum of that era.

ONTARIO DEPARTMENT OF EDUCATION

Annual Examinations 1903 — ARITHMETIC — High School Entrance

Note: One mark is to be deducted for each misspelt word. A maximum of five marks may be deducted for want of neatness.

1. (a) Divide the sum of $\dfrac{16}{\frac{7}{15} \text{ of } 2\frac{8}{11} \text{ of } \frac{11}{35}}$ and $\dfrac{4\frac{1}{2} \text{ of } 5\frac{1}{4} + 2\frac{3}{8}}{4\frac{1}{2} \text{ of } 5\frac{1}{4} - 10\frac{5}{8}}$

(15) by the difference between $\dfrac{2\frac{1}{5} \text{ of } 2\frac{1}{2}}{\frac{3}{14} \text{ of } 3\frac{2}{5}}$ and $\dfrac{3\frac{1}{4} + 4\frac{1}{3}}{6\frac{1}{2} + 1\frac{1}{12}}$

(15) (b) Multiply 352.128 by 41.40625 and divide 4.03046721 by .0729.

2. (25) A farmer sells to a merchant 3015 lbs. of hay at $16.00 a ton and receives in payment 8 lbs. of tea at 60¢ a lb., 13 lbs. of coffee at 45¢ a lb., 66 lbs. of sugar at 24 lbs. for a dollar, 65 lbs. 7 oz. of raisins at 10 2/3¢ a lb., 14 lbs. 13 oz. of bacon at 16¢ a lb., and the balance in cash. How much cash did the farmer receive?

3. (25) The average selling price of steel rails was $25.33 1/3 a ton and the profit at that price was 6%. If the total profits for the year were $36,024.00, how many tons were sold?

4. (25) At $17.00 per M., what would be the cost of the lumber required to build a sidewalk 1/8 of a mile long, 8 ft. wide and 2 in. thick, supported by three continuous lines of scantling 4 in. square?

5. (25) A grocer buys 80 lbs. of tea at 21¢ a lb. and mixes it with a dearer tea he has on hand. He sells the mixture at 35¢ a lb. and receives altogether $43.75, clearing $15.25 on the whole. How many lbs. of the dearer tea did he use and what was its cost per lb.?

6. (20) The interest on $840 for 1 2/5 years is $58.80. Find the interest on $650 for two years at the same rate.

7. (25) A railway passenger counts the telegraph posts on the line as he passes them. If they are 58 2/3 yeards apart and the train is going 48 miles an hour, how many will he pass per minute?

8. If a cubic foot of water weighs 1000 oz., how many tons of water are there in a reservoir 53 ft. 4 in. long, 16 ft. 6 in. wide and 20 ft. deep?

2.1 Mathematics Education 1900-1949 (cont'd.)

The examination shows a heavy emphasis on the performance of computation in a problem solving context. Since the problems involve manipulation and conversion of imperial units there is a predominence of common fraction arithmetic. Mixed numbers are used quite extensively while decimal fractions play a relatively minor role. As we move in the late seventies toward metric units, the contrast between our present curricula and the curricula of that era becomes even more striking. The problems on the 1903 examination which were relevant to the social and business needs in 1903 (and perhaps even in 1949), do not match the needs of contemporary society and would be regarded as somewhat artificial. (Who calculates interest over a period of 1 2/5 years or specifies distances between telegraph poles as 58 2/3 yards?) Furthermore, the hand calculator has eliminated the need for detailed computations such as 352.128 ÷ 41.40625. These differences might seem superficial until we observe that the use of common fractions rather than decimal fractions in a problem changes dramatically the arithmetic skills called into play in the solution of that problem.

A comparison of the 1903 examination with contemporary mathematics curricula shows the former to be rather demanding and yet restricted in its scope. There is no preliminary algebra, no probability or statistics, no integers, and no geometry. The problem solving involved is rather limited with little opportunity for creative thinking. Obviously, those skills we define as the "basics" have changed dramatically during this century in response to changing sociological and technological demands.

There was very little change in the format of instructional materials in mathematics between 1900 and 1949. Since economics did not usually permit the purchase of class sets of texts, there was often only one text and this was used by the teacher. Problems were copied by the teacher onto the blackboard. The students would transcribe these problems into their notebooks where they would write out orderly solutions. The instructional sections of mathematics texts were generally addressed to the teacher and were didactic, colorless passages with relatively high reading levels and no motivational content. Since most students would not encounter algebra or analysis in their schooling and since the development of computational skill was the main objective of the mathematics program, the "why" of arithmetic operations was subordinated to the "how". The mathematics text began as a cookbook of mathematical recipes for performing specific algorithms and solving specific problems.

2.1 Mathematics Education 1900-1949 (cont'd.)

The main function of testing was to assess student achievement and provide the information necessary to promote or fail the student at each level. Examinations were therefore a screening device. For this reason it was important that the examination generate a performance bell curve which would indicate unsatisfactory performance for a designated proportion of students and satisfactory performance for those remaining. Hence, all examinations were normed. Though class tests were administered periodically, most tests were cumulative and evaluation was summative.

2.2 Mathematics Education 1950-1969

After the Second World War, science and technology came to the fore. This war, more than any before it, had been a matching of technologies. Such devices as radar, incendiary bombs, submarines and V-2 rockets had been born of science and had increased man's awareness of the power of technology. The atomic bomb with its awesome destructive powers brought man to the realization that he was indeed living in a technological age. Military conquests would now be won in the laboratory - the country with the greatest technology would have the greatest security. And so, this line of thinking intensified the technological competition between the U.S.S.R. and the United States referred to as the "Cold War".

News of the Russian launching of Sputnik on October 4, 1957, swept like a tidal wave across this continent leaving in its wake an abiding fear of vulnerability and inadequacy. The reverberations of this shock wave were felt most strongly in the educational community. The professional mathematicians and scientists who had been denouncing the "rote skills" approach to mathematics and advocating a more conceptual approach, gained the ear of the American Government. They argued, with some justification, that the late introduction of subjects such as algebra and calculus was retarding the development of young American scientists who lagged two or more years behind their European and Soviet counterparts in their mathematical training. (Bob Hope observed, shortly after the Sputnik launch, "This all proves that the Russians have better German scientists than we have."). The American government commissioned a group of professional mathematicians and mathematics educators to renovate the elementary mathematics curriculum. From this renovation the so-called "new math" was born.

The necessity for computational skill had been significantly diminished by the proliferation of cash registers, adding machines, computers and other such devices. The rationale for mathematics had shifted from the support of daily business to the support of modern technology. Hence, elementary

2.2 Mathematics Education 1950-1969 (cont'd.)

school mathematics was seen as providing the foundation for the more formal mathematics of high school and ultimately university. The structures of number systems were investigated and with them came the terminology of modern algebra "commutativity", "associativity", "identity" and "inverse element". Congruence arithmetic, residues, set theory, bases other than ten and the number-numeral distinction were all introduced into the elementary school curriculum.

Between 1960 and 1969 a host of new mathematics texts began to appear. The instructional sections of these texts were expanded in scope and depth. Authors took pains to develop lessons in a manner designed to elicit complete understanding of underlying concepts. Usually, these passages addressed both teacher and student though the instructions were ostensibly for the student. Class sets of texts were now quite commonplace and the assumption was that students would learn much of their mathematics from the text itself. Rote drill was significantly reduced in texts and pages of computational exercises were replaced by pages of exploration which introduced and developed concepts in logical sequence. The traditional division algorithm was replaced with the so-called subtractive method, designed to build on the child's understanding of division as a form of repeated subtraction. Number lines and printed tables reduced the child's need to memorize the basic number facts. Arithmetic equations were written horizontally and with brackets as a prelude to the algebraic formalization which was to follow.

The increasing orientation of texts to student use brought with it increased decorative design. Simple non-functional yet appealing art decorated the pages of these new texts. The old black and white texts gave way at first to two color art and then to four color art in alternate signatures. By 1967 the major basal texts such as Modern School Mathematics (Houghton-Mifflin) were four color throughout.

The dimensions of mathematics texts increased during this period with the inclusion of more topics, more content and decorative art. Additional topics in the mathematics curriculum and the new demand for pedagogical strategies generated a need for a teacher's edition. The introduction of the teacher's edition was an important landmark in the evolving role of the mathematics text. Through the teacher's edition the author could address specific and parenthetical comments to the teacher, thereby relieving the student text of inappropriate copy. Most authors, however, did not exploit this opportunity to the fullest during this era. They continued to fill the pages of the text with a high density of print. The instructions though detailed were often inaccessible to the student simply because the page appeared too formidable. (see Discovering Mathematics, 1968, Charles E. Merrill Publishing Company).

2.2 Mathematics Education 1950-1969 (cont'd.)

During this period testing continued to be predominately summative. Standardized tests with local or national norms gained in popularity and use. By the mid-to-late sixties the foundations for continuous testing, modular learning and criterion-based evaluation were already in existence but such approaches to testing were not common practice. By the end of the sixties the content and instructional materials in elementary mathematics had sustained radical transformations though for the most part classroom management, instruction and evaluation procedures were relatively unchanged.

2.3 Mathematics Education in the Seventies

The late sixties saw the dawn of disillusionment in the new math approach. Sets, transformations, mappings and base 8 arithmetic brought few observable dividends while computational skills were seen to have diminished significantly. Public awareness of reduced computational competence in the students of the new math triggered a back-to-basics backlash.

Simultaneous to this revolution in mathematics curriculum, there occurred a revolution in educational methodology. Learning theories such as those of Piaget, Bruner, Dienes and others introduced a new movement in pedagogy. Educators began to look at such things as small group instruction, individualization, extensive manipulative activity and alternative instructional modalities.

During the so-called golden years of the sixties, the market for mathematics texts had grown to extraordinary proportions. Whereas mathematics texts had formerly enjoyed lifetimes in excess of 10 years, the highly competitive market had reduced this span to 3 to 5 years. The Addison-Wesley series Elementary School Mathematics dominated the K-6 mathematics market in the late sixties and early seventies with a highly acceptable balance of skills and concept development. Addison-Wesley followed this series in 1973 with Investigating School Mathematics. This series while financially successful, did not attain the same market dominance as its predecessor. The lesson format employed an investigatory approach to mathematical concepts followed by an appropriate amount of practice and consolidation of ideas. Though this series was pedagogically sound by present standards, it lost a significant portion of the market to the Scott-Foresman program Mathematics Around Us (copyright 1975). The latter program embodied a winning combination of mathematical presentation and visual appeal. High quality art, graphics and photography such as first evidenced in the ill-fated Field Mathematics Program put Mathematics Around Us in a dominant market position.

2.3 Mathematics Education in the Seventies (cont'd.)

Publishers had come to realize the importance of the so-called "flip" test in the marketing of texts. That is, a text when scanned by an evaluator should display esthetically pleasing pages in terms of layout and design. Such esthetics relate not only to graphic design but also to print density or "white space". A page which is print-dominant discourages reading, while a page displaying large areas of white space promises easy mastery and invites student participation. The teacher in flipping through a text is usually looking for an appropriate proportion of pages presenting open blocks of arithmetic exercises and few instructions. If a text gives immediate evidence of a computation orientation mixed with some graphic esthetics and motivational layout it passes the flip test.

The recent demand for white space and low print density might be attributed to the increasingly prominent role of television in the lives of children today. The typical family orientation to visual rather than print media mitigates against materials demanding high reading comprehension and toward pictoral presentation. More than ever, readability tests are applied to new mathematics texts and those which show high reading levels or high print densities suffer severe setbacks in the market place. The SRA program, Mathematics Learning System (1974) in spite of some good instructional ideas failed to realize its potential share of the basal market because its print density was too high and the apparent lack of computational pages made it appear deficient in skill development.

The Macmillan program Series M (copyright 1976) on the other hand was extremely successful because it displayed pages and pages of arithmetic exercises, generous proportions of white space and high quality graphics. This openness imparted to the series the aura of back-to-basics skill development. This orientation coincided with the peak of the back-to-basics movement and won for the program some crucial state adoptions and significant market penetration.

At least four major publishers of mathematics texts have released new programs in 1978. The Addison-Wesley series Math in Our World has responded to the increased emphasis on layout, design and white space with an extremely attractive and pedagogically sound set of texts. This is the first major American mathematics program to employ metric units exclusively and to develop a decimal approach to arithmetic. Scott-Foresman, Houghton-Mifflin and SRA are also launching new programs and they promise to acquire significant shares of the market.

The emerging commitment to continuous progress and individualized instruction had far reaching implications for

2.3 Mathematics Education in the Seventies (cont'd.)

instructional materials. In particular, the role of the basal text changed in the face of a demand for materials more conducive to individualized instruction and grouping. One response to the need for a variety of materials tailored to individual learning styles was the mathematics lab. Among the first mathematics labs was the Individually Prescribed Instruction Program (IPI-Math) developed at the University of Pittsburgh. This lab consisted of a set of instructional booklets keyed to 359 behavioral objectives.

The Continuous Progress Laboratory (CPL lab) published by Educational Development Corporation (E.D.C.) in the early '70's was a set of activity cards containing pre and post tests keyed to a specific set of skill objectives. These cards were correlated to all the major basal programs so that a student requiring additional instruction in a particular objective would be referred by the correlation to the relevant section in whatever text he was using. The pre-test/post test format coupled with the correlations to basal programs laid the foundation for a management system approach to instruction.

Math labs and kits flourished during the early seventies. Companies such as SRA and E.D.C., developed extensive product lines in kits and shared a substantial share of the instructional materials market. Such kits usually consisted of modular instructional units, activity cards and/or cassettes. The advantage of this format resided in the greater flexibility it afforded the teacher in channelling students at different learning stages into different instructional activities. While the kit embodied a format more consistent with individualized instruction its superior flexibility was bought at a price. The price of flexibility was increased demand on the teacher for activity prescription and record keeping. For this reason the kit was usually relegated to a supportive role in the instructional materials repertoire.

The last few years have seen more widespread adoption of an instructional objectives approach to mathematics curriculum. Consonant with this development is a wider acceptance of criterion-referenced testing and the creation of instructional management systems. Recently, E.D.C. has developed such a computer-based management system called ICRT Basics +. This program contains 384 instructional objectives spanning a grade 1-8 mathematics curriculum. Students are placed at an appropriate level in the program and administered a test booklet covering a module of 8 objectives. The booklets are scored by a computer in Tulsa and the student is directed by a personalized printout to specific sections in the appropriate text for remediation or further instruction. Parallel tests are

2.3 Mathematics Education in the Seventies (cont'd.)

available for subsequent administration to measure learning. Of prime importance to the teacher is the Grouping Summary print-out which groups students by area of weakness for further instruction. When used effectively, this type of management system actually makes it possible for a teacher to conduct a highly individualized program without the burdensome manual record keeping.

At present, Michigan State is developing such a management system called MTSS (Michigan Teachers Support System). This program will require a computer terminal in each school to process the tests and although this may involve significant front-end costs, these costs might be outweighed by the accessibility and rapid turnaround offered by such a system. As costs of computer hardware continue to decrease, such management systems will become more feasible and will likely be in widespread use by school boards.

2.4 A Tabular Summary

This table summarizes some of the major changes in mathematics programs between 1900 and the present.

	Content	Instructional Materials	Evaluation
1900-1949	arithmetic skills	little or none (teacher text)	achievement tests
1950-1969	mathematical concepts	a basal text	norm-referenced tests
1970 ⟶	mathematical skills and concepts	basal text kits cassettes filmstrips	criterion-referenced texts

While it might appear that the back-to-basics trend is bringing the mathematics program full circle, it is clear from the table that, in fact, the opposite is true. The new math had a profound and irrevocable effect on the mathematics curriculum. Geometry, probability and statistics, measurement and preliminary algebra have now become an integral part of the K-8 program. The realm of instructional materials has exploded from the single text to a highly sophisticated complex of kits, cassettes, calculators and computers. Testing has evolved from the haphazard summative test to a tight criterion-referenced instrument with diagnostic and prescriptive capability.

3. Future Directions

It is difficult to predict future developments in instructional materials for so much is contingent upon the social and technological forces which will impinge upon education in the next few years. If extrapolation of the present trend toward increased accountability in education is valid, then it is reasonable to assume that state-wide or at least board-wide instructional objectives will specify the mathematics to be taught in each school. (Most States now have such objectives in mathematics). Furthermore, state and local jurisdictions will naturally move toward a criterion-referenced instrument to evaluate student performance relative to these objectives. With decreasing costs of computer hardware the transition to a computerized management system such as described in section 2.3 would seem inevitable.

The use of correlations through the computer will solve to a large extent the management problems associated with the mathematics lab. While the basal text will continue to be the main vehicle for large group instruction, the computer will direct individual students to specific cassettes, filmstrips or print materials for special instruction. Furthermore, C.A.I., will ultimately play a prominent role in computational skill development.

The importance of computational skill will continue to diminish as the hand calculator is more widely used. There will be, however, a much greater need for number sense, estimation skills and applications of mathematics to real-world problems. Of paramount importance is the present trend toward problem solving. Creative problem solving will emerge as the single most important topic in the mathematics curricula of the 1980's. Various problem solving techniques will be developed in the contexts of arithmetic, geometry, statistics and combinatorics. With these changes in content, instructional materials and evaluative techniques, the student of the future will emerge from elementary school mathematics armed with the analytical tools he will need to meet the challenge of an even more sophisticated world than the one in which we live.

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